

GLOBAL OPTIMIZATION

THEORY, ALGORITHMS, AND APPLICATIONS

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This volume contains a thorough overview of the rapidly growing field of global optimization, with chapters on key topics such as

- complexity,
- heuristic methods,
- derivation of lower bounds for minimization problems, and
- branch-and-bound methods and convergence.

The final chapter offers both benchmark test problems and applications of global optimization, such as finding the conformation of a molecule or planning an optimal trajectory for interplanetary space travel. An appendix provides fundamental information on convex and concave functions.

Global Optimization is intended for Ph.D. students, researchers, and practitioners looking for advanced solution methods to difficult optimization problems. It can be used as a supplementary text in an advanced graduate-level seminar.

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