

# Common Sense Mathematics

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*Published and Distributed by*  
The Mathematical Association of America

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In this first chapter we learn how to think about questions that need only good enough answers. We find those answers with quick estimates that start with reasonable assumptions and information you have at your fingertips. To make the arithmetic easy we round numbers drastically and count zeroes when we have to multiply.

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In real life there are few naked numbers. Numbers usually measure something like cost, population, time, speed, distance, weight, energy or power. Often what's measured is a rate, like miles per hour, gallons per mile, miles per gallon, dollars per gallon, dollars per euro or centimeters per inch.

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