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# Fuzzy Solution Concepts for Non-cooperative Games

Interval, Fuzzy and Intuitionistic Fuzzy  
Payoffs

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