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FOUNDATIONS OF QUANTUM Second Edition CHROMODYNAMICS

**An Introduction to Perturbative Methods
in Gauge Theories**

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